

# $V_{cb}$ and $V_{ub}$ CKM Matrix Elements

OMITTED FROM SUMMARY TABLE

See the related review(s):

Semileptonic  $B$  Hadron Decays, Determination of  $V_{cb}$  and  $V_{ub}$

## $V_{cb}$ MEASUREMENTS

For the discussion of  $V_{cb}$  measurements, which is not repeated here, see the review on “Determination of  $|V_{cb}|$  and  $|V_{ub}|$ .”

The CKM matrix element  $|V_{cb}|$  can be determined by studying the rate of the semileptonic decay  $B \rightarrow D^{(*)} \ell \nu$  as a function of the recoil kinematics of  $D^{(*)}$  mesons. Taking advantage of theoretical constraints on the normalization and a linear  $\omega$  dependence of the form factors ( $F(\omega)$ ,  $G(\omega)$ ) provided by Heavy Quark Effective Theory (HQET), the  $|V_{cb}| \times F(\omega)$  and  $\rho^2$  ( $a^2$ ) can be simultaneously extracted from data, where  $\omega$  is the scalar product of the two-meson four velocities,  $F(1)$  is the form factor at zero recoil ( $\omega=1$ ) and  $\rho^2$  is the slope, sometimes denoted as  $a^2$ . Using the theoretical input of  $F(1)$ , a value of  $|V_{cb}|$  can be obtained.

“OUR EVALUATION” is an average using rescaled values of the data listed below. The average and rescaling were performed by the Heavy Flavor Averaging Group (HFLAV) and are described at <https://hflav.web.cern.ch/>. The averaging/rescaling procedure takes into account correlations between the measurements.

### $|V_{cb}| \times F(1)$ (from $B^0 \rightarrow D^{*-} \ell^+ \nu$ )

VALUE	DOCUMENT ID	TECN	COMMENT
<b><math>0.03561 \pm 0.00043</math> OUR EVALUATION</b>			with $\rho^2=1.205 \pm 0.026$ and a correlation 0.338. The fitted $\chi^2$ is 30.2 for 23 degrees of freedom.
<b><math>0.0355 \pm 0.0008</math> OUR AVERAGE</b>			Error includes scale factor of 1.7. See the ideogram below.
$0.03483 \pm 0.00015 \pm 0.00056$	<sup>1</sup> WAHEED	19	BELL $e^+ e^- \rightarrow \Upsilon(4S)$
$0.0359 \pm 0.0002 \pm 0.0012$	<sup>2</sup> AUBERT	09A	BABR $e^+ e^- \rightarrow \Upsilon(4S)$
$0.0392 \pm 0.0018 \pm 0.0023$	<sup>3</sup> ABDALLAH	04D	DLPH $e^+ e^- \rightarrow Z^0$
$0.0431 \pm 0.0013 \pm 0.0018$	<sup>4</sup> ADAM	03	CLE2 $e^+ e^- \rightarrow \Upsilon(4S)$
$0.0355 \pm 0.0014 \begin{smallmatrix} +0.0023 \\ -0.0024 \end{smallmatrix}$	<sup>5</sup> ABREU	01H	DLPH $e^+ e^- \rightarrow Z$
$0.0371 \pm 0.0010 \pm 0.0020$	<sup>6</sup> ABBIENDI	00Q	OPAL $e^+ e^- \rightarrow Z$
$0.0319 \pm 0.0018 \pm 0.0019$	<sup>7</sup> BUSKULIC	97	ALEP $e^+ e^- \rightarrow Z$
● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●			
$0.0346 \pm 0.0002 \pm 0.0010$	<sup>8</sup> DUNGEL	10	BELL Rep. by WAHEED 19
$0.0359 \pm 0.0006 \pm 0.0014$	<sup>9</sup> AUBERT	08AT	BABR Repl. by AUBERT 09A
$0.0344 \pm 0.0003 \pm 0.0011$	<sup>10</sup> AUBERT	08R	BABR Repl. by AUBERT 09A
$0.0355 \pm 0.0003 \pm 0.0016$	<sup>11</sup> AUBERT	05E	BABR Repl. by AUBERT 08R
$0.0377 \pm 0.0011 \pm 0.0019$	<sup>12</sup> ABDALLAH	04D	DLPH $e^+ e^- \rightarrow Z^0$

0.0354 ± 0.0019 ± 0.0018	<sup>13</sup> ABE	02F BELL	Repl. by DUNGEL 10
0.0431 ± 0.0013 ± 0.0018	<sup>14</sup> BRIERE	02 CLE2	$e^+e^- \rightarrow \Upsilon(4S)$
0.0328 ± 0.0019 ± 0.0022	ACKERSTAFF	97G OPAL	Repl. by ABBIENDI 00Q
0.0350 ± 0.0019 ± 0.0023	<sup>15</sup> ABREU	96P DLPH	Repl. by ABREU 01H
0.0351 ± 0.0019 ± 0.0020	<sup>16</sup> BARISH	95 CLE2	Repl. by ADAM 03
0.0314 ± 0.0023 ± 0.0025	BUSKULIC	95N ALEP	Repl. by BUSKULIC 97

<sup>1</sup> Uses fully reconstructed  $D^{*-} \ell^+ \nu$  events ( $\ell = e$  or  $\mu$ ) and  $\eta_{EW} = 1.0066$ .

<sup>2</sup> Obtained from a global fit to  $B \rightarrow D^{(*)} \ell \nu_\ell$  events, with reconstructed  $D^0 \ell$  and  $D^+ \ell$  final states and  $\rho^2 = 1.22 \pm 0.02 \pm 0.07$ .

<sup>3</sup> Measurement using fully reconstructed  $D^*$  sample with a  $\rho^2 = 1.32 \pm 0.15 \pm 0.33$ .

<sup>4</sup> Average of the  $B^0 \rightarrow D^*(2010)^- \ell^+ \nu$  and  $B^+ \rightarrow \bar{D}^*(2007) \ell^+ \nu$  modes with  $\rho^2 = 1.61 \pm 0.09 \pm 0.21$  and  $f_{+-} = 0.521 \pm 0.012$ .

<sup>5</sup> ABREU 01H measured using about 5000 partial reconstructed  $D^*$  sample with a  $\rho^2 = 1.34 \pm 0.14^{+0.24}_{-0.22}$ .

<sup>6</sup> ABBIENDI 00Q: measured using both inclusively and exclusively reconstructed  $D^{*\pm}$  samples with a  $\rho^2 = 1.21 \pm 0.12 \pm 0.20$ . The statistical and systematic correlations between  $|V_{cb}| \times F(1)$  and  $\rho^2$  are 0.90 and 0.54 respectively.

<sup>7</sup> BUSKULIC 97: measured using exclusively reconstructed  $D^{*\pm}$  with a  $a^2 = 0.31 \pm 0.17 \pm 0.08$ . The statistical correlation is 0.92.

<sup>8</sup> Uses fully reconstructed  $D^{*-} \ell^+ \nu$  events ( $\ell = e$  or  $\mu$ ).

<sup>9</sup> Measured using the dependence of  $B^- \rightarrow D^{*0} e^- \bar{\nu}_e$  decay differential rate and the form factor description by CAPRINI 98 with  $\rho^2 = 1.16 \pm 0.06 \pm 0.08$ .

<sup>10</sup> Measured using fully reconstructed  $D^*$  sample and a simultaneous fit to the Caprini-Lellouch-Neubert form factor parameters:  $\rho^2 = 1.191 \pm 0.048 \pm 0.028$ ,  $R_1(1) = 1.429 \pm 0.061 \pm 0.044$ , and  $R_2(1) = 0.827 \pm 0.038 \pm 0.022$ .

<sup>11</sup> Measurement using fully reconstructed  $D^*$  sample with a  $\rho^2 = 1.29 \pm 0.03 \pm 0.27$ .

<sup>12</sup> Combines with previous partial reconstructed  $D^*$  measurement with a  $\rho^2 = 1.39 \pm 0.10 \pm 0.33$ .

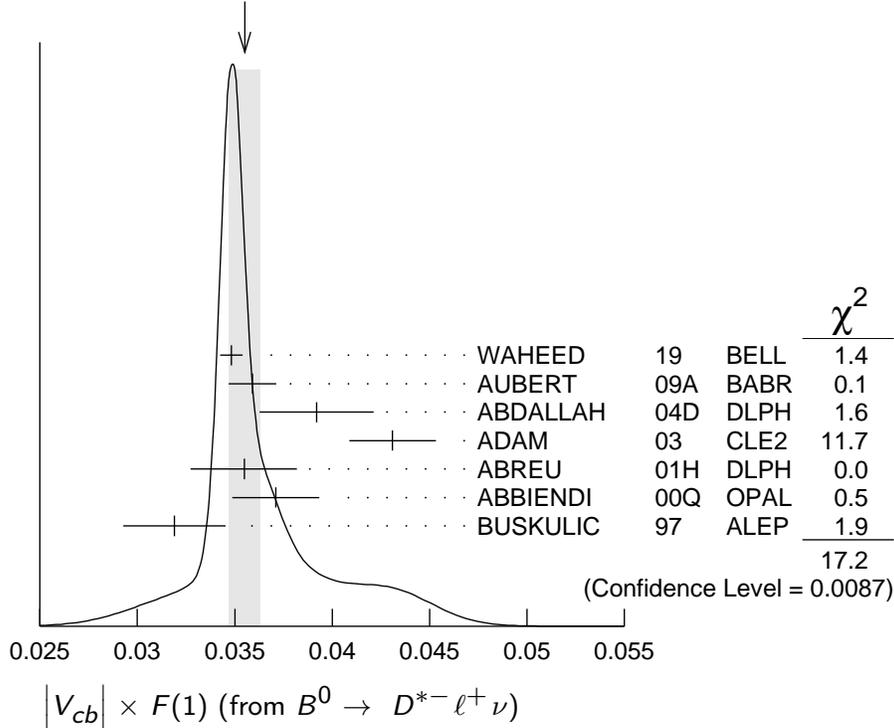
<sup>13</sup> Measured using exclusive  $B^0 \rightarrow D^*(892)^- e^+ \nu$  decays with  $\rho^2 = 1.35 \pm 0.17 \pm 0.19$  and a correlation of 0.91.

<sup>14</sup> BRIERE 02 result is based on the same analysis and data sample reported in ADAM 03.

<sup>15</sup> ABREU 96P: measured using both inclusively and exclusively reconstructed  $D^{*\pm}$  samples.

<sup>16</sup> BARISH 95: measured using both exclusive reconstructed  $B^0 \rightarrow D^{*-} \ell^+ \nu$  and  $B^+ \rightarrow D^{*0} \ell^+ \nu$  samples. They report their experiment's uncertainties  $\pm 0.0019 \pm 0.0018 \pm 0.0008$ , where the first error is statistical, the second is systematic, and the third is the uncertainty in the lifetimes. We combine the last two in quadrature.

WEIGHTED AVERAGE  
 $0.0355 \pm 0.0008$  (Error scaled by 1.7)



**$|V_{cb}| \times G(1)$  (from  $B \rightarrow D^- \ell^+ \nu$ )**

VALUE	DOCUMENT ID	TECN	COMMENT
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**$0.04157 \pm 0.00100$  OUR EVALUATION** with  $\rho^2 = 1.128 \pm 0.033$  and a correlation 0.751.

The fitted  $\chi^2$  is 4.7 for 8 degrees of freedom.

**$0.0422 \pm 0.0010$  OUR AVERAGE**

$0.04229 \pm 0.00137$	<sup>1</sup> GLATTAUER	16 BELL	$e^+ e^- \rightarrow \Upsilon(4S)$
$0.0423 \pm 0.0019 \pm 0.0014$	<sup>2</sup> AUBERT	10 BABR	$e^+ e^- \rightarrow \Upsilon(4S)$
$0.0431 \pm 0.0008 \pm 0.0023$	<sup>3</sup> AUBERT	09A BABR	$e^+ e^- \rightarrow \Upsilon(4S)$
$0.0416 \pm 0.0047 \pm 0.0037$	<sup>4</sup> BARTELT	99 CLE2	$e^+ e^- \rightarrow \Upsilon(4S)$
$0.0278 \pm 0.0068 \pm 0.0065$	<sup>5</sup> BUSKULIC	97 ALEP	$e^+ e^- \rightarrow Z$

• • • We do not use the following data for averages, fits, limits, etc. • • •

$0.0411 \pm 0.0044 \pm 0.0052$	<sup>6</sup> ABE	02E BELL	Repl. by GLATTAUER 16
$0.0337 \pm 0.0044 \begin{smallmatrix} +0.0072 \\ -0.0049 \end{smallmatrix}$	<sup>7</sup> ATHANAS	97 CLE2	Repl. by BARTELT 99

<sup>1</sup> Obtained from a fit to the combined partially reconstructed  $B \rightarrow \bar{D} \ell \nu_\ell$  sample while tagged by the other fully reconstructed  $B$  meson in the event. Also reports fitted  $\rho^2 = 1.09 \pm 0.05$ .

<sup>2</sup> Obtained from a fit to the combined  $B \rightarrow \bar{D} \ell^+ \nu_\ell$  sample in which a hadronic decay of the second  $B$  meson is fully reconstructed and  $\rho^2 = 1.20 \pm 0.09 \pm 0.04$ .

<sup>3</sup> Obtained from a global fit to  $B \rightarrow D^{(*)} \ell \nu_\ell$  events, with reconstructed  $D^0 \ell$  and  $D^+ \ell$  final states and  $\rho^2 = 1.20 \pm 0.04 \pm 0.07$ .

<sup>4</sup> BARTELT 99: measured using both exclusive reconstructed  $B^0 \rightarrow D^- \ell^+ \nu$  and  $B^+ \rightarrow D^0 \ell^+ \nu$  samples.

<sup>5</sup> BUSKULIC 97: measured using exclusively reconstructed  $D^\pm$  with a  $a^2 = -0.05 \pm 0.53 \pm 0.38$ . The statistical correlation is 0.99.

- <sup>6</sup> Using the missing energy and momentum to extract kinematic information about the undetected neutrino in the  $B^0 \rightarrow D^- \ell^+ \nu$  decay.
- <sup>7</sup> ATHANAS 97: measured using both exclusive reconstructed  $B^0 \rightarrow D^- \ell^+ \nu$  and  $B^+ \rightarrow D^0 \ell^+ \nu$  samples with a  $\rho^2 = 0.59 \pm 0.22 \pm 0.12_{-0}^{+0.59}$ . They report their experiment's uncertainties  $\pm 0.0044 \pm 0.0048_{-0.0012}^{+0.0053}$ , where the first error is statistical, the second is systematic, and the third is the uncertainty due to the form factor model variations. We combine the last two in quadrature.

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## $V_{ub}$ MEASUREMENTS

For the discussion of  $V_{ub}$  measurements, which is not repeated here, see the review on "Determination of  $|V_{cb}|$  and  $|V_{ub}|$ ."

The CKM matrix element  $|V_{ub}|$  can be determined by studying the rate of the charmless semileptonic decay  $b \rightarrow u \ell \nu$ . The relevant branching ratio measurements based on exclusive and inclusive decays can be found in the  $B$  Listings, and are not repeated here.

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## $V_{cb}$ and $V_{ub}$ CKM Matrix Elements REFERENCES

WAHEED	19	PR D100 052007	E. Waheed <i>et al.</i>	(BELLE Collab.)
GLATTAUER	16	PR D93 032006	R. Glattauer <i>et al.</i>	(BELLE Collab.)
AUBERT	10	PRL 104 011802	B. Aubert <i>et al.</i>	(BABAR Collab.)
DUNGEL	10	PR D82 112007	W. Dungenl <i>et al.</i>	(BELLE Collab.)
AUBERT	09A	PR D79 012002	B. Aubert <i>et al.</i>	(BABAR Collab.)
AUBERT	08AT	PRL 100 231803	B. Aubert <i>et al.</i>	(BABAR Collab.)
AUBERT	08R	PR D77 032002	B. Aubert <i>et al.</i>	(BABAR Collab.)
AUBERT	05E	PR D71 051502	B. Aubert <i>et al.</i>	(BABAR Collab.)
ABDALLAH	04D	EPJ C33 213	J. Abdallah <i>et al.</i>	(DELPHI Collab.)
ADAM	03	PR D67 032001	N.E. Adam <i>et al.</i>	(CLEO Collab.)
ABE	02E	PL B526 258	K. Abe <i>et al.</i>	(BELLE Collab.)
ABE	02F	PL B526 247	K. Abe <i>et al.</i>	(BELLE Collab.)
BRIERE	02	PRL 89 081803	R. Briere <i>et al.</i>	(CLEO Collab.)
ABREU	01H	PL B510 55	P. Abreu <i>et al.</i>	(DELPHI Collab.)
ABBIENDI	00Q	PL B482 15	G. Abbiendi <i>et al.</i>	(OPAL Collab.)
BARTELT	99	PRL 82 3746	J. Bartelt <i>et al.</i>	(CLEO Collab.)
CAPRINI	98	NP B530 153	I. Caprini, L. Lellouch, M. Neubert	(BCIP, CERN)
ACKERSTAFF	97G	PL B395 128	K. Ackerstaff <i>et al.</i>	(OPAL Collab.)
ATHANAS	97	PRL 79 2208	M. Athanas <i>et al.</i>	(CLEO Collab.)
BUSKULIC	97	PL B395 373	D. Buskulic <i>et al.</i>	(ALEPH Collab.)
ABREU	96P	ZPHY C71 539	P. Abreu <i>et al.</i>	(DELPHI Collab.)
BARISH	95	PR D51 1014	B.C. Barish <i>et al.</i>	(CLEO Collab.)
BUSKULIC	95N	PL B359 236	D. Buskulic <i>et al.</i>	(ALEPH Collab.)

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